Dust and Time:
On Relativity Theory and the Reality of Time.

(Claudio Calosi)

"The only reason for time
is so that everything does not happen at once"

(A.Einstein).

It is often claimed that Relativity theory, both the special and the general[1], have refuted Kant's trascendental idealism. Godel (1949a) is a notable exception. In this short, beautiful and rather mysterious work Godel in fact argues that relativity theory support the kantian thesis[2] of the ideality of time, i.e *the ontological thesis that time in itself is unreal.* In what follows I will address exactly this question: the relation between relativity theory and the reality of time as understood by Godel. The structure of the paper is rather simple. In section 1 I will reconstruct[3] Godel's original argument. From this reconstruction it will be clear that the argument cries for an interpretation. The following sections will be devoted then to such tentative interpretations[4]. I will group them under two headings, the *modal interpretation* and the *epistemic interpretation* that will be addressed in section 2 and 3 respectively[5]. Finally in section 4 I will give some conclusive remarks.

1. Godel's Argument for the Unreality of Time.

Godel's argument has an elegant structure. It can be reconstructed along the following lines.

1.1) Preliminary. A necessary condition that has to be met in order to conclude that time is real is singled out.
1.2) Argument from STR. Time as described by STR can not meet the requirement in i), therefore time in STR is not real.
1.3) Argument from general solutions to Einstein's field equations[6] in GTR. The argument from STR in ii) can be overcome passing to cosmological models that represent general solutions to EFE.
1.4) Argument from new solutions to EFE. There exist different cosmological solutions to EFE where the necessary requirement singled out in i) can not be met. Therefore, as in ii), time in these models is not real.

1 STR and GTR respectively from now on.
2 We need to be more precise. In the first critique Kant argues for the empirical reality and trascendental ideality of space and time. The trascendental ideality of time is purported to show that time is just a form of our intuition, i.e time in itself is not real. This is what is meant by Godel when he talks about the ideality of time. However unreality and ideality are two ontological distinct thesis in the original kantian framework. It is not possible to spend too much time on these niceties here. Since the paper will deal with Godel's understanding of this question they will be leaved aside, and we will think about ideality as *just* an unreality thesis.
3 My analysis will be a reconstruction rather than a simple presentation. I will, for example, recast the original argument in more modern terms.
4 I will try to construct the more robust interpretations I can think of. For this reason they will be close but different from the ones found in literature.
5 I will leave aside another tentative interpretation, namely the symmetry interpretation, due to Gordon Belot. This is a very fascinating suggestion indeed, but as Belot himself points out, read in this way Godel's argument does not cut really deep. I think that this is not what Godel's himself had in mind but I personally consider it a very interesting starting point for a new argument altogether. See Belot (2005) in the references.
6 EFE from now on.

1.5) The step. Different cosmological models in iii) and iv) differ only for contingent non lawlike features. Since these new models represent physical possibilities time is not real in all of the possible models allowed by the theory.

I now turn to give a detailed analysis step by step.

1.1) Preliminary. A necessary condition for the reality of time.

If Time is real change is real[7]. And change becomes possible only trough an *objective lapse of time*. The existence of an objective lapse of time is equivalent to the fact that *reality consists of an infinity of layers of now which comes into existence successively*[8]. Here Godel singles out an informal criterion[9] time has to meet in order to be real. The task is to formulate such a criterion in the formal language of relativity theory. It is this task that I take up now. A brief construction of a hierarchy of topological, metrical, and temporal[10] conditions is needed.

A *relativistic spacetime is a pair* $<M, g_{ab}>$ where M is a *n-dimensional differentiable manifold* and g_{ab} is *a Lorentz signature metric*, i.e a signature (1,n-1)[11], defined on all of M. This is the most general geometric structure GTR assigns to the world. In the limiting case of vanishing gravitational fields we have Minkowski spacetime $<M, \eta_{ab}>$ where η_{ab} is the so called Minkowski tensor that assigns a flat non euclidean geometry to M. Minkowski spacetime is the geometric structure STR assigns to the world instead[12].

$<M,g_{ab}>$ is said to be *temporally orientable* iff it admits a global time sense, i.e a possible way to distinguish locally between a future and a past direction at every point $p \in M$. A sufficient condition for temporal orientability is the existence of a non vanishing timelike vector field on M, i.e an assignment of a vector μ to each point in M such that $\mu^a\mu_a > 0$[13]. It is a basic lemma that for any $<M, g_{ab}>$ where M is simply connected than $<M,g_{ab}>$ is temporally orientable.

Let $<M,g_{ab}>$ be a temporally orientable spacetime. Choose one of the two possible orientation as giving the future direction of time[14]. Then for every $p, q \in M$, *p chronologically precedes q,* written formally as $p << q$, iff there is a future directed timelike curve from p to q. The chronological past of p is $I^-(p) := \{q \in M : q << p\}$[15]. Again it is a basic lemma that $<<$ is a *transitive* relation. We require that $<<$ is also *irreflexive*, i.e for every $p \in M$ $\neg (p<<p)$, then $<M,g_{ab,}$

7 This premise is not explicit in Godel (1949a) but it is in the preliminary drafts, namely Godel (1946-49 b2-c1). As far as I know Godel has never questioned its validity.

8 Note that this picture is almost verbatim the picture of a growing block derived from Broadian notion of Becoming, i.e it is not an A-theoretic notion as described by McTaggart's A series. The identification of lapse of time with the image of a growing block is again never questioned by Godel.

9 Naturally this criterion can be questioned. However it is not completely arbitrary. There are good reasons to adopt it, derived in particular from McTaggart's argument and from the arguments in the metaphysical exposition of the concept of time in Kant's first critique. It is not possible to deal with these niceties here.

10 This hierarchy of conditions can be pushed further adding *causal* conditions.

11 I adopt the sign convention (+, -,-,-). The Lorentzian metric induces the famous classification of vectors into timelike, spacelike and null.

12 The problem of conventionality of metrical geometry is neglected. A necessary and sufficient condition for M to admit a Lorentzian metric is the existence of a non vanishing line element field on M. There are certain topological properties M have to satisfy in order to define such line element field. Any non-compact M will do but not every compact M will do. For example in the case of dim (M) =2 and signature (1,1) a sphere will not do.

13 Here $\mu^a\mu_a = \mu^1\mu^1 - \mu^2\mu^2 - \mu^3\mu^3 - \mu^4\mu^4$ and naturally dim (M) = 4. The so called Einstein's summation convention is used.

14 At this level this is a pure conventional choice. The argument developed later in the section depends crucially on time orientability, i.e on the possibility of distinguishing past and future directions but it does not hinge upon the choice of such a distinction as the one corresponding to the one that obtains, if at all, in the world. I will write a temporally oriented spacetime as $<M, g_{ab}, \uparrow>$.

15 The chronological future is defined analogously.

$\uparrow>$ is said to have a *time order*[16]. Now everything is in place to recast Godel's informal criterion into the formal language of relativity theory[17].

Let $\lambda : I \rightarrow M$, where I is an arbitrary interval, be a smooth timelike curve and consider the tangent space T_p at a point $p \in \lambda$. Let H_p be the three dimensional spacelike submanifold orthogonal to T_p[18]. H_p is the M-submanifold that contains simultaneous events. It is then natural to identify the "layers of now" of Godel's informal criterion with the different spacelike hypersurfaces orthogonal to the tangent space at a point. Consider now two different hypersurfaces at two different points p_1 and p_2. According to Godel's informal criterion they have to "come into existence successively". It is easy to recast this requirement using the formal language developed above. H_{p2} comes into existence successively to H_{p1} iff $p_1 << p_2$ for every p_1 and p_2 belonging respectively to H_{p1} and H_{p2}. Moreover this coming successively into existence has to represent the *objective lapse of time*.
Let then $t: M \rightarrow R$ be a differentiable map assigning time coordinates to events in M in such a way that $t_{(p1)} < t_{(p2)}$ iff H_{p2} comes into existence successively to H_{p1}. The differentiable map just defined is called a *global time function*. To represent an objective lapse of time we impose two different requirements, i) that it has to be invariant under the symmetries of the $<M, g_{ab}>$ considered and ii) that if two different global time function are defined they agree on the relations of temporal succession assigned. We can make these assertion precise: Here's a formulation for both:

i) Let $\varphi : M \rightarrow M$ be an isometry[19] of $<M,g_{ab}>$. We require that if $t_{(p1)} < t_{(p2)}$, then $t_{\varphi(p1)} < t_{\varphi(p2)}$.
ii) Let t and t' be two different maps $t,t': M \rightarrow R$ defined using different hypersurfaces. Then for every p_1 and p_2 such that $t_{(p1)} < t_{(p2)}$ we have $t'_{(p1)} < t'_{(p2)}$.

The construction of this global time function seems to capture Godel's informal criterion for the reality of time. In fact it can be used to represent an objective lapse of time and it encodes the fact that reality consists of layers of now that come into existence successively. Thus the result of the argument above can be stated easily. The *possibility of defining a global time function on* $<M,g_{ab},\uparrow>$ that is invariant under the symmetries of the cosmological model into consideration *is a necessary condition for the reality of time.*

1.2) The Argument from STR.

Framed this way the question becomes quite straightforward. Is it possible to define a global time function on Minkowski spacetime that is invariant under its symmetries? The answer is no. Let me give first an informal argument following Godel's own[20]. The well known result of relativity of simultaneity imply that a foliation of Minkowski spacetime into spacelike hypersurfaces of simultaneity is observer dependent, i.e different hypersurfaces are singled out by different non parallel timelike lines[21]. Since we have identified these hypersurfaces of simultaneity with the layers of now this result amounts to say that every observer has her own set of "now". Moreover, due to the relativity principle that establishes the equivalence of every observer, there is no way to regard some of the hypersurfaces of simultaneity as representing the objective lapse of time over some others.

16 If $<<$ is *reflexive* then $<M,g_{ab}>$ is said to be *chronologically vicious*.
17 Note that it has been talked only of a time order and not of time coordinates. It will not be difficult however to do so.
18 It is easy to prove that it has to be spacelike, i.e for every $\mu \in H_p$, $\mu^a\mu_a < 0$.
19 An isometry is a symmetry that preserves the metric structure.
20 In Godel (1949a) the argument from STR is just three lines: " But if simultaneity is something relative (...) reality can not be split up into such layers in an objectively determined way. Each observer has his own set of "nows", and none of these various systems of layers can claim the prerogative of representing the objective lapse of Time".
21 These curves model the worldlines of possible observers.

Let me give a more formal argument.
Let L, L' be two frames, i.e some collection of maximal timelike parallel lines[22] in Minkowski time oriented spacetime $\langle M,\eta_{ab}, \uparrow\rangle$[23] and let L_1 and $L_{1'}$ be two timelike lines belonging to L and L' respectively that intersect at p. It is a basic fact of Minkowski spacetime that the hypersurfaces of simultaneity at p relative to L_1 and $L_{1'}$, call them H_{p1} and $H_{p1'}$, will be different. Use the different hypersurfaces to construct two different differentiable maps t and t' respectively according to the procedure described in 1.1).

Now the problem is to see whether t and t' meet i) and ii). Temporal orientability becomes crucial in Minkowski spacetime at this stage of the argument. It in fact guarantees that both t and t' meet 1)[24]. Thus they are serious candidate to represent the objective lapse of time. The serious problem arise with ii). It is in fact always possible to find a point $q \in M$[25] such that $t_{(p)} < t_{(q)}$ but $t'_{(q)} < t'_{(p)}$. This does not settle the question by itself. It can be in fact argued that even if both t and t' meet requirement i) it is possible to single out one of them as the one representing the true objective lapse of time in terms of the geometric structure of Minkowski spacetime. And here's the last step. There is always an isometry $\varphi : M \rightarrow M$ that maps L_1 into L_1'[26]. This means that it is not possible to single out a timelike line as a privileged one in geometric terms. Thus it is not possible to define an *invariant global*[27] *time function* over $\langle M,\eta_{ab},\uparrow\rangle$. According to the argument in 1.1) then *Time, as described by STR, is not real*[28].

1.3) The Argument from general solutions to EFE in GTR.

Suppose we define a new structure over $\langle M,\eta_{ab},\uparrow\rangle$, namely a congruence C of inertial timelike curves[29]. This structure would define over Minkowski spacetime a privileged inertial worldline through each spacetime point. We call $\langle M,\eta_{ab},\uparrow, C\rangle$ augmented Minkowski spacetime. It would be possible to take the family of spacelike hypersurfaces everywhere orthogonal to C which is transverse, i.e every hypersurface intersects each inertial worldline exactly once. We could use this family of hypersurfaces to define a global time function t as described in 1.1). It can be proved that this function is the only one that is invariant under the symmetries of the augmented Minkowski

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22 I'm using lines instead of curves for sake of simplicity. The argument will go trough anyway dealing with timelike curves but we will have to take into consideration the tangent space at every point and the spacelike submanifold orthogonal to that space.

23 The arrow stands for a timelike vector field defined over M in order to distinguish past and future direction. Minkowski spacetime is always time orientable, i.e such a vector field is always definable. This will play a crucial role at a certain stage of my argument. Time orientability will fail in several general relativistic spacetimes but not in the ones I will deal with in the paper.

24 Temporal orientability becomes crucial here because otherwise a map φ: $M \rightarrow M$ that is a reflection about an orthogonal subspace of an arbitrary timelike line will be an isometry of the not time oriented Minkowski spacetime. Then let p and q be two points such that $p \ll q$. By our construction it has to be the case that $t_{(p)} < t_{(q)}$. But it is always possible to find an isometry of the type described above such that it leaves p fixed and maps q to q', i.e $\varphi(p) = p$ and $\varphi(q) = q'$, such that $t_{(q')} < t_{(p)}$. Hence the global time function t will not be invariant under the symmetries of the spacetime considered.

25 It has to be spacelike separated from p.

26 This is a way to capture nicely in terms of the four dimensional minkowskian geometry the *relativity principle*.

27 In fact only a partial time ordering that is invariant under the symmetries of Minkowski spacetime can be defined, namely for timelike separated points.

28 Note that this is due to the very geometric structure of Minkowski spacetime. According to the argument in 1.1) if the world would have been newtonian then Time would have been real. The reader is urged to find an argument herself. As an hint consider the fact that simultaneity hypersurfaces are not singled out by the orthogonality criterion. An interesting objection to this line of thought is considered by Godel himself in a footnote. I will deal with it section 2.2).

29 We use curves instead of lines since it will be easier to generalize the result to general relativistic spacetimes. Strictly speaking there are no inertial timelike curves in Minkowski spacetimes that are not timelike lines.

spacetime. According to the argument in 1.1) Time in <M,η_{ab},↑, C> would then be real[30]. Godel himself hinted at this possibility in a footnote. The problem is the definition of C is contrary to the spirit itself of STR since it amounts to reintroduce a privileged reference frame[31].
But consider now a general solutions to EFE in GTR. Or better, consider the so called dust[32] solutions in which spacetime is filled with dust motes representing galaxies in free fall motion, i.e the only force acting on dust motes is given by the gravitational field. A solution to dust cosmologies is given by a spacetime manifold M, a spacetime geometry encoded by the metric tensor g_{ab}, and a function describing the dust density at each point and a congruence C' of timelike geodesics representing the dust motes of the galaxies. It is possible to prove that the last two structures can be reconstructed in terms of the components of the metric tensor g_{ab} alone. It seems then that the geometry of some general relativistic spacetimes itself provides the additional structure[33] needed to run the argument given for the augment Minkowski spacetime. It will be only necessary to replace the congruence C of inertial timelike lines with the congruence C' of dust worldlines and then proceed geometrically to construct the family of spacelike hypersurfaces orthogonal to the tangent vector field of the dust worldlines at every point and then define an invariant global time function over <M, g_{ab}>. We just need to check if the global time function defined geometrically is furthermore invariant under the symmetries of the cosmological model considered, since these symmetries usually differ from the symmetries of the augmented Minkowski spacetime. But a general solution to EFE *usually admits no non trivial spacetime symmetries, hence the global time function is trivially invariant*. Then, according to the argument in 1.1) *time in general dust solutions to EFE is real*[34]. The reality of time seems thus regained, even if only at a cosmological level, passing to GTR.

1.4) The Argument from new solutions to EFE in GTR.

Note that the argument in 1.3) is a two step argument.

i) Firstly the spacelike submanifold, i.e the set of spacelike hypersurfaces everywhere orthogonal to the tangent space of the congruence of timelike curves is constructed.
ii) Then the set of spacelike hypersurfaces singled out in i) is used to construct an invariant global time function t: M → R on the basis of the relation of chronological precedence << that capture the requirement of coming into existence successively of that very family of hypersurfaces.

Godel was able to find new solutions to EFE that render *both the steps impossible*. Here's a brief formal representation of Godel's cosmological model. I will call it the *R-Universe for rotating universe*[35]. <M,g_{ab}, T^{ab}> is a solution to EFE with positive cosmological constant.

iii) R-Universe is dust filled[36] and the dust is everywhere rotating, i.e $\nabla_{[a}V_{b]} \neq 0$.

30 Actually this conclusion will not follow that easily. Remember in fact that the condition singled out in 1.1) is a just a *necessary condition and not a sufficient one too*. However the map t will be a natural candidate to represent the objective lapse of time in augmented Minkowski spacetime.
31 In terms of four dimensional geometry this will mean that there is no isometry mapping the set of timelike curves defined by C into another arbitrary set of timelike curves.
32 These solutions are the ones Godel deals with.
33 In Godel's own words "the existence of matter, however, as well as the particular kind of curvature of spacetime produced by it, largely destroy the equivalence of different observers and distinguish them conspicuously from the rest, namely those which follow the mean motion of matter".
34 Again, see footnote 30.
35 The reason is quite obvious.
36 Formally this means that the stress energy tensor T^{ab} is given by $T^{ab}= \rho V^aV^b$ where ρ is the density of the dust and V^a is the four velocity field of the dust.

iv) $M = R^4$[37]. Thus M is simply connected and it is a basic lemma that any simply connected M is time orientable.
v) There does not exist a single global time slice, i.e spacelike hypersurfaces without edges.
vi) The spacetime is chronologically vicious[38]. Thus for every point $p \in M$ there is a closed timelike curve, i.e a timelike curve whose tangent vectors at every point are always timelike and future directed which comes back to p.

It is now easy to prove that the argument in 1.3) and summarized in i) and ii) above, fails in the R-universe. It turns out that *it is possible to take orthogonal subspaces to the worldlines of matter iff the matter is everywhere non rotating*, i.e $\nabla [_aV_b] = 0$[39]. But naturally, *this is not possible in R-Universe, given iii)*. It is even possible to prove a stronger result. In fact orthogonality is not the only candidate to define a 3 dimensional spacelike submanifold that intersect a given a timelike curve exactly once. It is possible to consider for example *homogeneity*, i.e to consider an hypersurface where the distribution of matter has the same value for density and pressure, or hypersurfaces with *minimal possible curvature.* The problem in the R-Universe is that, given v) *there are simply no spacelike hyprsurfaces whatsoever that are transeverse*. This argument shows that i) fails. Another simple argument establishes that even ii) fails. It in fact follows from vi) that for every $p \in M$, $p \ll p$ holds. This *contradicts the requirement of being irreflexive imposed* in 1.1) *on the relation of chronological precedence.* And it is this very relation that is used to define a possible candidate for a global time function. These arguments taken together imply that is not possible to define an invariant global time function in the R-Universe. Thus, according again to the argument in 1.1) *time in the R-Universe is not real*.

1.5) The Step.

Call the cosmological model described by the general solutions to EFE of section 1.3) G-Universe, for general universe. The arguments in the last two sections seem to establish that time is real in the G-Universe but unreal in the R-Universe. But R-Universe is just a physically possible universe. Moreover our actual universe is similar to the G-Universe[40]. We have all kind of empirical evidence that the actual universe is not a R-Universe. Now, the unreality thesis is an ontological thesis of certain importance if it is referred to our actual universe and not to just a possible one. In the very last part of the paper Godel tackles exactly this point in a rather mysterious way. Here's a possible reconstruction of his last argument.

i) R-Universe is a physically possible universe since it is a solution to EFE and it satisfy other additional requirements[41].
ii) R-Universe can not be ruled out a priori because of its causal structure[42].
iii) Time is unreal in the R-Universe, by the argument in 1.4).
iv) The main difference between G-Universe, .i.e our actual universe and the R-Universe is the non lawlike contingent difference in the distribution of matter and its motion.
v) It is *unsatisfactory* to maintain that an ontological difference of such importance, namely the

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37 Godel's cosmological model is topologically very simple. R^4 is in fact the simplest topology for M.
38 See footnote 16.
39 This is a special case of the so called Frobenius' theorem. It has been argued that Godel was pushed towards the discovery of his solutions exactly by this technical point. Only in a second time he realized the presence of the closed timelike curves that strengthen the argument. See Malament (1995).
40 For the purpose of the paper I will assume that our universe, described by the so called Robertson Friedman Walker solutions, or RFW, is a G-Universe.
41 For example the so called energy condition.
42 I will take for granted its validity for sake of the argument.

existence of time, depends solely upon the contingent features in iv)[43].

vi) ∴ *Time is unreal also in the actual universe, a G-Universe*.

This concludes, almost with Godel's own words, the reconstruction of the argument presented in Godel (1949a). It is not difficult to see from this reconstruction that the argument cries for an interpretation. Naturally the argument in 1.5) seems to carry the heavier burden of every interpretation, especially the rather mysterious premise v). At first sight we usually do make such "unsatisfactory" claims and consider them quite natural. For example we normally find claims like "Space in our Universe is open but have the mass distribution been different it would have been closed" unproblematic. What then assign a special status to the claim assessing that the lapse of time, and consequently its existence, according to Godel, depends on the mass distribution[44]? I will now turn to the task of providing tentative interpretations to account for the strong thesis in vi).

2. Godel's argument revisited: the Modal Interpretation.

As I have already pointed out most of the burden for every tentative interpretation seems to be the argument presented in section 1.5). The starting point for a modal interpretation of the argument is the recognition of its *modal nature:* a conclusion, namely the unreality of time, that is established for a *possible* world, is extended to the *actual* one. Hence it is an argument that deals with the classical *modal gap between possibility and actuality*. I can think of three different versions for a modal reading. I will call them the kripkean version, the broadian version and the kantian version for reasons that will be obvious.

2.1) A kripkean[45] version.

Suppose we endorse an essentialist view of this form: for every entity there is a set of properties P_i all of which any entity of that kind must have in order to be real. Call it the *essentialist assumption*. And suppose furthermore that we accept Kripke's famous argument for the necessity of identity statements. Then a claim like "E = the entity which has the set of properties P_i" is a necessary truth, i.e it has to hold in all possible worlds. Call it the *kripkean assumption*. With this background in hand is possible to run an interesting version of Godel's modal argument. Here's a possible construction:

i) Time = that entity that lapses[46].
ii) Since the statement in i) is an identity statement it has to hold in all possible worlds.(from kripkean assumption)
iii) In all possible worlds Time has to lapse in order to be real (from i), ii) and the essentialist assumption).

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43 In Godel's own words " The mere compatibility with the laws of nature of worlds in which there is no distinguished absolute time [...] throws some light on the meaning of time also in those worlds in which an absolute time *can* be defined. For, if someone asserts that this absolute time is lapsing, he accepts as a consequence that, whether or not an objective lapse of time exists depends on the particular way in which matter and its motion are arranged in the world. *This is not a straightforward contradiction; nevertheless a philosophical view leading to such consequences can hardly be considered as satisfactory*" (First italic original, second italic mine).

44 This is the central point of what can be called, following Belot, Earman's challenge.

45 This is to be taken as a suggestion. It has no intention to be an adequate reading of Kripke's main argument in *Naming and Necessity*. See Kripke, S. (1972), *Naming and Necessity*, Harvard University press.

46 Or better "Time = that entity which has this essential set of properties: $P_1, P_2, ..P_l$ = it lapses, ...P_n". I think there are good reasons to hold i) even if personally I would be inclined not to endorse it. Godel surely held it. I think that it is possible to argue for i) if two sorts of considerations are taken into account, namely kantian arguments from the metaphysical exposition of the concept of time and Mc Taggart's argument. Note that both are explicitly mentioned by Godel himself. It is not possible to enter in those details here.

iv) There is a physically possible universe in which time does not lapse, namely the R-Universe. (by the argument in 1.4)
v) Time does not exist in all possible worlds. (from i), iii) and iv)).
vi) ∴*Time is unreal in the actual world* (a fortiori).

Put more informally if lapsing is an essential property of time, a property time must have in all possible worlds in order to be real according to the modal variant of the essentialist assumption, then if it is possible to find a world in which time does not lapse, for example the R-Universe, you have shown that time is not real. This argument uses an essentialist strategy to fulfill the modal gap that was problematic in the original formulation of section 1.5). The price to pay for such an argument is however quite high. Firstly it has to be argued for premise i). Even if an essentialist position is endorsed an argument for i) is still needed. As I have noted in footnote 46 there are reasons to hold i) but the question of whether lapsing is an essential property of time is far from settled[47]. But apart from that it is the modal variant of the essentialist assumption itself that seems an heavy metaphysical load.

2.2) A Broadian[48] Version.

Recall the argument in 1.1) and the one from STR in 1.2). It could be argued that the second invariant condition in 1.1) on the different global time functions, the one telling that two different maps have to agree on the assignments of time coordinates, is too strong a requirement. Following this lead the argument from STR does not establish that time is not real *but only that the objective lapse of time has to be relativized to a particular timelike line*. In fact given an arbitrary timelike line and a point on that line there is just one spacelike submanifold orthogonal to the line that contains such a point. Taking the family of the spacelike hypersurfaces at different points on the line it is possible to define a global time function that is invariant under the symmetries of time oriented Minkowski spacetime. Godel himself imagined this possible line of thought and in a footnote he gave a response. Briefly the response goes along these lines. The notion of time's lapse, as it is constructed in section 1.1, is closely related to the one proposed by Broad in response to McTaggart's argument[49], and *is equivalent to a change in existing*. Thus a relativization of the notion of lapsing of time means in the end a relativization of the notion of existence. But the *concept of existence can not be relativized without destroying its meaning completely*, using Godel's words almost verbatim. I think that this discussion for the argument in STR can be developed in an argument to fill the modal gap in 1.5). I now turn to give such an argument.

i) Lapse of time means a change in existing (by the construction in 1.1))
ii) Reality of time means a change in existing (by the argument in 1.1) reality of time is linked to its lapse)
iii) A relativization of the notion of time's lapse means a relativization of the concept of existence (by i)).
iv) A relativization of the reality of time means a relativization of the concept of existence (by ii) and iii)).
v) Reality of time has to be relativized to a certain mass distribution (by the arguments in 1.3 and 1.4).
vi) The concept of existence has to be relativized to a certain mass distribution (by iv) and v)).
vii) The concept of existence can not be relativized without destroying its meaning completely

47 It is actually denied in all of the so called B- theories of time.
48 See Broad, C.H. (1923), *Scientific thought*, Cambridge University press.
49 See Mc Taggart, J.E. (1908), *The Unreality of time*, Mind, **18**. It is not possible to deal with their positions here. A contemporary advocate of a broadian notion of becoming is Michael Tooley. See Tooley, M. (1997), *Time, Tense, Causation*, Clarendon press.

(by Godel's *assumption*).

viii)Reality of time destroys the meaning of the concept of existence completely (by v) and vii)).

ix) ∴*Time is unreal*.

This argument uses the concept of relativization to overcome the modal gap in 1.5). The first thing to note is a weakness of the argument as it stands. The original godelian remark from STR was intended to refer to a certain notion of relativization, relativization to different observers. Premise v) uses the notion of relativization to a certain mass distribution instead. At first sight it seems difficult to argue that these two notions are the same or even that they entail the same consequences, at leats it seems to me. But apart from that most of the burden of the argument is Godel's assumption vii). It is probably rooted in the platonic-leibnizean metaphysics Godel endorses also in his mathematical work. But the same remark of the last section applies here: the endorsement of this metaphysical background seems a high price to pay.

2.3) A kantian[50] version.

As I have briefly pointed out in footnote 2 Godel's original argument was intended to show that relativity theory provides support for the *ideality of time, not only its unreality*. The ontological thesis of the ideality of time Godel had in mind was originally defended by Kant in the Trascendental Aesthetic. Thus it seems not completely inadequate to look back at Kant in order to find a possible interpretation for the argument in 1.5). A few words are needed. In the metaphysical exposition of the concept of time Kant argues for two different thesis, the first being the a priority thesis, i.e a thesis according to which the representation of time is given a priori, and the second being the intuition thesis, i.e the thesis according to which the representation of time is not a concept but an intuition[51]. Each of these thesis is supported with two different arguments[52]. For the purpose of the paper the important one is the second argument for the a priority thesis[53]. Kant argues that it is not possible, in respect to appearances, remove the representation of time itself though it is quite possible to have a representation of time that is void of appearances. Let's try to construct an argument using these remarks.

i) Existence of time has to be relativized to a certain mass distribution (from the arguments in 1.3) and 1.4)).
ii) This means that if there would be no matter there would be no lapse of time, and hence time would not be real (from i) and the link between reality of time and its lapse in 1.1)).
iii) But according to the second argument from the metaphysical exposition of the concept of time we would have the representation of time even without appearances, i.e even without masses (from Kant's Trascendental Aesthetic).
iv) Hence, existence of time can not be relativized to a certain mass distribution (from iii).
v) Reality of time is then an absolute matter, independent from the contingent presence of masses, i.e if it is real is then real in all possible worlds that differ only for mass distribution (from iv).
vi) There is a world, namely the R-Universe, in which Time is not real (by the argument in 1.4).
vii) Time is not real in all possible worlds that differ from R-Universe only for the contingent feature of mass distribution (from v) and vi)).
viii)Our Universe, G-Universe, differ from the R-Universe, just for the mass distribution (from

50 For Kant I have used Kant, I. Translation by N.K.Smith. (1929), *Critique of pure reason*, St.Martin's press.

51 Kant's terminological choices can sometimes cause confusion to the reader. For example the section's title is Metaphysical Exposition of the *concept* of time but one of its fundamental thesis is that *time is not a concept*.

52 Kant's scholars are divided on this point. Some of them read the arguments as two different logically independent arguments, some other as a single two step argument. I will not deal with these exegetical points here.

53 Precisely the argument is found in B46.2

the argument in 1.4).

ix) ∴*Time is unreal also in our Universe* (from vii) and viii)).

I have to admit that I do not find the argument convincing in this form although it seems at least persuasive. But, differently from the argument for the kripkean version this is not only due to the fact that you have to buy a controversial assumption, namely the result from the second argument for the a priority thesis in iii). It is the passage from i) to ii) that seems problematic. The fact that existence of time has to be relativized to a certain mass distribution does not imply by itself that without any masses there would not be lapse of time. Take a possible cosmological model, i.e a possible solution to EFE. It could be the case that there were no masses and yet the solution attributes to the world a geometric structure that allow to define a global time function to describe the objective lapsing of time. There are in fact solutions to EFE, namely De Sitter solutions, where $T^{ab}= 0$ and yet given a point on a timelike curve it is possible to define the spacelike submanifold orthogonal to the tangent vector at that point. However, before concluding the section, I want to point out three considerations in order to render the argument more plausible. Firstly, consider for example De Sitter solutions. It is true that the *components of the stress energy tensor vanish but the components of the metric tensor g_{ab} do not*. And it is still an open question whether we should consider the metric itself in GTR as carrying energy. If so, since energy and mass are intimately related in relativity theory, it would be difficult to sustain that De Sitter solutions represent a world that is void of appearances, if we are to use Kant's notion of appearance. Secondly, it could be argued that generally speaking the dynamical laws of GTR are formulated in such a way in which space, time and spacetime crucially depend on the existence of matter distribution. *It is just in some solutions, and not in the dynamical equations themselves, that the components of the stress energy tensor could vanish*. And, at last, and probably more importantly, in such cosmological models there are symmetries mapping timelike curves into one another[54]. There is no way to single out a privileged congruence of timelike curves and then use the spacelike submanifold orthogonal at every point of the curves to define an invariant global time function. Recall in fact that in the argument in 1.3) a privileged congruence was singled out as being the one representing those observers that were comoving with the cosmic matter. *But here we are working under the hypothesis in which there is no matter at all*. Naturally if there is no cosmic matter no observers can be singled out as comoving with it.

These are the stronger modal variants of the argument in 1.5) I can think of. This exposition is in no way intended to be exhaustive. Something somehow striking has to be underlined though. Every version presented here seems to *depend crucially on the assumption of some very controversial claims*, namely the kripkean modal variant of the essentialist assumption[55] in 2.1), the platonic-leibnizean assumption of the impossibility of relativizing the notion of existence in 2.2), and the kantian assumption derived from the metaphysical exposition of the concept of time in 2.3). It is for this reason that it could seem promising to adopt a different strategy altogether that does away with all of them. The epistemic interpretation of the next section can be considered an example of such a strategy.

3. Godel's argument revisited: the Epistemic Interpretation.

The overall strategy of the previous section was to find some ontological assumption that could be used to fill the modal gap between the conclusions obtained for a physically possible world and the

54 Actually just timelike geodesics, but this difference is playing no role here.

55 I have called them assumptions because I have accepted them without any arguments. But naturally in their original framework they are theoretical conclusions rather than assumptions. They remain however very controversial conclusions.

actual one. The strategy of this section hinges upon some epistemic claims about the experiences of observers in the different worlds. It draws mainly from this passage of Godel:

"For, in whatever way one may assume time to be lapsing, there will always exist, possible observers to whose experienced lapse of time no objective lapse corresponds. But, if the experience of lapse of time can exist without an objective lapse of time, no reason can be given why an objective lapse of time should be assumed at all".

The passage deals with the experience of lapse of time. Consider this experience in the G-Universe. I will call it G-experience of time. As time lapses in the G-Universe different layers of now comes into existence successively, by the argument in 1.1). Hence experiencing time lapsing is equivalent to *experiencing a change in existing*, *namely a change that is consistent with the relation of chronological precedence* since it is this very relation that is used to formalized the notion of coming into existence successively in the informal argument of section 1.1).
Take now a step back to the mathematical representation of the R-Universe. Godel himself introduces a global spacetime coordinate system with a global time coordinate[56], global time that orders the events on each worldline of matter consistently with the relation of chronological precedence, due to the technical fact that Godel's spacetime is time orientable. Consider now the experience of an observer in the R-Universe that is comoving with a worldline of cosmic matter. The mathematical representation above is supposed to model such an experience, and it is exactly the same representation given in the G-Universe. This imply that any observer that is comoving with a worldline of matter will experience a change in existing, and furthermore a change that is consistent with the relation of chronological precedence as defined for points on that worldline itself. R-Experience of time is *thus indistinguishable from G-experience of time if we are restricted to non closed timelike curves*[57]. Call this argument the *Indistinguishibility argument*. Within this framework is possible to run an interesting epistemic argument, at least in a weak version[58]. It follows roughly these lines.

i) G-Universe and R-Universe are physically possible universes according to GTR because they both represent solutions to the dynamical equations of the theory.
ii) *It is possible to have the same experience of lapse of time in both the universes in i)* (by the indistinguishibility argument above).
iii) Time does not lapse in R-Universe (by the argument in 1.4).
iv) *It is possible to have the phenomenological experience of lapse of time* even if nothing corresponds to it in reality (by ii) and iii)).
v) There are two reasons to endorse the thesis that time is really lapsing, the ones derived from our best spacetime theories, namely GTR, and the ones derived from the phenomenology of our experience.
vi) GTR provides no ground to endorse that time is lapsing (by i) and iii)).
vii) The phenomenology of our experience provides no ground to endorse that time is lapsing (by iv)).
viii) We have no reason to believe that time is lapsing (by v), vi), vii)).
ix) ∴ *We have no reason to believe that time is real* (by viii) and the link between reality of time and its lapse in 1.1)).

I think this is a very interesting argument. There are immediately some important remarks to make.

56 This renders possible the definition of a global time function.
57 That means not only for worldlines for observers that are comoving with the cosmic matter. It is then possible to calculate the difference in the assignment of time coordinates for every non closed timelike curves and regard the needed corrections as deviation due to the mass distribution.
58 It will be clear later the reason why I have called this variant, a weak version of an epistemic argument.

The conclusion of this epistemic interpretation is far weaker that the one reached at the end of the modal interpretation. In fact, if one of the modal variants presented in the previous section were successful it would establish that *time is unreal*. The epistemic argument, even if successful would just establish that we *have no reasons to believe that time is real*. The overall strategy of the epistemic interpretation could be adequately described as *shifting the burden of the proof* to the realist[59]. Before concluding the section I want to hint at a possible way to strengthen the argument. In this weak version the indistinguishibility argument is the basis to establish premise ii), i.e that *it is possible to have the same phenomenological experience* in both universes. It is however not enough to establish that the experience *is* the same. So, in iv) I have concluded that *it is possible to have the experience of lapse of time even if nothing corresponds to it* but I was *not* able to argue that *this is in fact the case*. A first step towards such an argument would have to show that we would experience that time is lapsing even if we were to be confined to a closed timelike curve in the R-Universe. I think that in this case there are considerations that point to different directions. Godel's spacetime is time orientable. This means that locally it is possible to distinguish at every point between the future direction and the past direction, even on a closed timelike curve. Then there is a sense in which locally we would experience a change in existing that is consistent with the relation of chronological precedence even on a closed timelike curve. This brief argument , if correct, would establish that *we do have an experience of lapse of time*, according to the construction in this section, *at least locally at every spacetime point in the manifold*. But the problem is when we consider the closed timelike curve as a whole. Let's consider what the experience of lapse of time is supposed to be more carefully. According to the construction in 1.1) as time lapses new layers of now comes into existence. As I have already pointed out this means that the experience of time lapsing is an experience of change in existing. But we can go further. It is a very particular kind of change in existing, it is a *growth* in existence[60]. Consider now this thought experiment. Take a timelike curve $\lambda : I \rightarrow M$ and two points p and q on λ such that $p<<q$. Suppose an observer were to follow that curve and count the sum of what's existing at p, call it S_p and at q, call it S_q. Then, for every arbitrary timelike curve in the G-Universe the observer will find that $S_p < S_q$. Hence an observer in the G-Universe will *always experience lapse of time as a growth in existence*. Take a closed timelike curve in R-Universe instead. The observer will find out that, at a critical radius, there will be two points p and q such that $p<<q$ but $S_q < S_p$. Moreover consider the situation at a single spacetime point p. Since in R-Universe there is always a closed timelike curve passing through p, and since for a closed timelike curve $p<<p$ holds, the observer, to experience lapse of time as a growth in existence, should find out that $S_p < S_p$. But this is clearly impossible. In the R-Universe, in the unlucky circumstance of being confined to a closed timelike curve, the observer 's experience of lapse of time will not always be that of a growth in existence. Thus it seems to follow from this argument that *even if it is possible* to have a G-experience of time, i.e an experience of time an inhabitant of a G-universe would have, in the R-Universe, *this is not always the case*. There are several ways to reply to such an argument. First, it could be argued that probably the inhabitants of the R-Universe are short living creatures like us so that they will probably never spend their entire life on a closed timelike curve. Second, it could be argued that my thought experiment does not go through since it implies the possibility of actually counting the sum of what's existing at every single spacetime point. But technically speaking relativity theory implies that there is no such notion of a spatially extended present. All these suggestions do not sum up to an argument but I think they are interesting as a possible starting point. The weak version of the epistemic argument presented above is however not touched by these considerations.

59 Both these two consequences are clearly stated in Savitt (1994). His reconstruction of the argument is however different in significant points.

60 This analysis is consistent with the picture of a growing block derived from the notion of broadian becoming.

4. Conclusion: remarks on Godel's remark.

I'd like to conclude with some final brief remarks. It is clear that the original version of Godel's argument is as beautiful as mysterious. Godel himself acknowledges that his own argument does not refute the thesis of the reality of time showing that it is contradictory but it rather makes it unsatisfactory. Naturally it all hinges upon the question of why it is unsatisfactory and to what extent it is so. In sections 2 and 3 I have tried to construct interpretations of the original argument that, if successful would at least clarify why and how the reality thesis is unsatisfactory. The modal interpretation implies that it is simply false, the epistemic interpretation suggests that arguments that are independent from GTR or the phenomenology of experience are needed.

But let's for a moment take for granted that some of the arguments in sections 2 and 3 are indeed successful and ask if they are enough to establish what they are supposed to. Strictly speaking they establish that a certain conception of time and becoming, namely the one that is captured by Godel's informal criterion, is not compatible with relativity theory. If furthermore it is assumed that relativity theory provides the best description of reality we have, it can be concluded that this very concept of time *does not refer to anything real,* i.e *time as intended in 1.1) is not real*. But the fact that time has that very particular structure is still a very controversial question. Many of the philosophers of time and probably most of the physicists would argue that lapsing is an unnecessary feature of time drawn from the mistaken perspective of common sense. And even among the ones defending the necessity of lapse there would be disagreement on how to describe that lapse itself. That said if successful, Godel's argument would at least establish that, using Weyl's[61] marvelous words “the objective world is, it does not happen”.

61 See Weyl, H. (1949), *Philosophy of mathematics and natural sciences*, Princeton University press. This passage has been called to my attention by John Earman.

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